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MATHEMATICAL ASSESSMENT OF CYBERSECURITY RISKS IN THE DIGITAL ECONOMY: A QUANTITATIVE APPROACH TO FINANCIAL SYSTEM PROTECTION

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Abstract. The rapid expansion of digital financial services creates both economic opportunity and systemic cybersecurity vulnerability. Quantifying the relationship between cyber-attack frequency, associated economic losses, and protective investment remains an open challenge in the literature. This paper introduces a straightforward mathematical framework — combining a loss function, an exponential risk-reduction model, and a return-on-security-investment (ROSI) formula — to measure and optimise cybersecurity spending in a digital economy context. The model is calibrated using empirical data from Uzbekistan's banking sector (2018–2023). Results show that a one-unit increase in the Cybersecurity Readiness Score (CRS) reduces expected annual losses by USD 3.4 million on average, and that the optimal security budget allocation lies between 8 % and 11 % of total IT expenditure. The framework offers financial institutions and regulators a transparent, data-grounded tool for evidence-based cybersecurity policy decisions.

Keywords: cybersecurity, digital economy, risk modelling, ROSI, financial systems, Uzbekistan, loss function, mathematical model.

МАТЕМАТИЧЕСКАЯ ОЦЕНКА КИБЕРБЕЗОПАСНОСТИ В ЦИФРОВОЙ ЭКОНОМИКЕ: КОЛИЧЕСТВЕННЫЙ ПОДХОД К ЗАЩИТЕ ФИНАНСОВОЙ СИСТЕМЫ

Аннотация. Стремительное расширение цифровых финансовых услуг создает как экономические возможности, так и системную уязвимость в области кибербезопасности. Количественная оценка взаимосвязи между частотой кибератак, связанными с ними экономическими потерями и защитными инвестициями остается нерешенной задачей в литературе. В данной статье представлена простая математическая модель, сочетающая функцию потерь, экспоненциальную модель снижения рисков и формулу рентабельности инвестиций в безопасность (ROSI), для измерения и оптимизации расходов на кибербезопасность в контексте цифровой экономики. Модель откалибрована с использованием эмпирических данных банковского сектора Узбекистана (2018–2023 гг.). Результаты показывают, что увеличение индекса готовности к кибербезопасности (CRS) на одну единицу снижает ожидаемые годовые потери в среднем на 3.4 млн долларов США, а оптимальное распределение

бюджета безопасности составляет от 8 % до 11 % от общих расходов на ИТ. Предложенная модель предоставляет финансовым институтам и регулирующим органам прозрачный, основанный на данных инструмент для принятия обоснованных политических решений в сфере кибербезопасности.

Ключевые слова: кибербезопасность, цифровая экономика, моделирование рисков, ROSI, финансовые системы, Узбекистан, функция потерь, математическая модель.

RAQAMLI IQTISODIYOTDA KIBERXAVFSIZLIK RISKLARINI MATEMATIK BAHOLASH: MOLIYA TIZIMINI HIMOYA QILISHGA MIQDORIY YONDASHUV

Annotatsiya. Raqamli moliyaviy xizmatlarning jadal kengayishi ham iqtisodiy imkoniyatlarni, ham tizimli kiberxavfsizlik zaifliklarini yuzaga keltiradi. Kiberhujumlar chastotasi, ular bilan bog'liq iqtisodiy zararlar va himoya investitsiyalari o'rtasidagi bog'liqlikni miqdoriy baholash ilmiy adabiyotlarda hamon dolzarb muammo bo'lib qolmoqda. Mazkur maqolada raqamli iqtisodiyot sharoitida kiberxavfsizlik xarajatlarini o'lchash va optimallashtirish uchun zarar funksiyasi, eksponentsial xavfni kamaytirish modeli va xavfsizlik investitsiyalari rentabelligi (ROSI) formulasini birlashtirgan oddiy matematik model taklif etilgan. Model O'zbekiston bank sektori (2018–2023) empirik ma'lumotlari asosida kalibrlangan. Natijalar shuni ko'rsatadiki, Kiberxavfsizlikka tayyorlik indeksi (CRS) bir birlikka oshganda, kutilayotgan yillik zararlar o'rtacha 3.4 million AQSh dollariga kamayadi va xavfsizlik budjetining maqbul taqsimoti umumiy AT xarajatlarining 8 foizidan 11 foizigacha bo'lgan qismini tashkil etadi. Ushbu model moliya institutlari va tartibga soluvchi organlarga kiberxavfsizlik siyosati bo'yicha asoslangan qarorlar qabul qilish uchun shaffof va ma'lumotlarga asoslangan vositani taqdim etadi.

Kalit so'zlar: kiberxavfsizlik, raqamli iqtisodiyot, risklarni modellashtirish, ROSI, moliya tizimlari, O'zbekiston, zarar funksiyasi, matematik model.

INTRODUCTION

Digital transformation has fundamentally altered the structure of modern economies. Online banking, electronic payment platforms, and cloud-based financial services have made capital flows faster, cheaper, and more accessible — yet they have simultaneously created new attack surfaces for cybercriminals. According to the International Monetary Fund, global financial-sector losses attributable to cyber incidents exceeded USD 12 billion in 2022, a figure nearly three times larger than five years earlier [1]. For emerging economies such as Uzbekistan, where the share of digital payments in total retail transactions grew from 21 % in 2018 to 73 % in 2023, the stakes are particularly high because protective infrastructure often lags behind adoption rates [2].

The academic literature on cybersecurity and economics has grown substantially over the past decade, yet a persistent gap remains: most studies either describe the problem qualitatively or rely on regression models that estimate past losses without providing actionable investment guidance. Gordon and Loeb [3] were among the first to formalise a security investment optimisation model, showing that firms should spend no more than 37 % of the expected loss on any single asset. Their model, however, was designed for a single firm and does not scale naturally to sector-level or national-level analysis. Subsequent extensions by Böhme and Schwartz [4] and Anderson et al. [5] added game-theoretic and insurance dimensions but kept the mathematical apparatus complex enough to limit practitioner uptake.

This paper proposes a simpler, transparent, and directly estimable mathematical framework suited to the needs of financial regulators and institutional risk managers in developing-economy settings. The core contribution is threefold. First, we define a composite Cybersecurity Readiness Score (CRS) that aggregates observable institutional indicators into a single index. Second, we specify an expected-loss function that links CRS to monetary damage in a mathematically tractable form. Third, we derive the optimal security budget share from a return-on-security-investment identity and validate all three components with six years of Uzbek banking-sector data. The remainder of the paper is organised as follows: the Methods section presents the mathematical model and data; the Results section reports parameter estimates, fitted loss curves, and optimal budget recommendations; the Discussion interprets findings in the policy context; and the Conclusion summarises contributions and limitations.

METHOD

The quantitative framework developed here rests on three building blocks: a Cybersecurity Readiness Score, an expected-loss function, and a return-on-security-investment optimisation condition. Each is described in turn, followed by an account of the data used for estimation.

Cybersecurity Readiness Score. Following established composite-index methodology [6], we define CRS as the equally weighted arithmetic mean of three normalised sub-indicators, each scaled to the interval [0, 1]:

Formula (1): $CRS = (1 / 3) * (TI + HR + RP)$

where TI denotes the Technology Infrastructure sub-index (network redundancy, encryption coverage, patch-management compliance), HR the Human Resources sub-index (share of staff with recognised cybersecurity certification, hours of annual training per employee), and RP the Regulatory and Policy sub-index (number of enacted cybersecurity standards relative to best-practice benchmark). Each sub-index is computed by min-max normalisation across the sample, so that the

institution with the highest observed value receives a score of 1 and the lowest receives 0.

Expected-Loss Function. Empirical studies consistently show that cyber-incident losses decline at a diminishing rate as protective measures improve — a relationship well captured by a negative exponential form [7]. We therefore specify the expected annual loss L (in USD millions) as:

Formula (2): L(CRS) = L0 * exp(-k * CRS)

where L0 > 0 is the baseline expected loss when CRS = 0 (no protective measures in place) and k > 0 is a decay parameter reflecting how quickly losses fall as readiness improves. Taking the natural logarithm of both sides yields the linear regression form:

Formula (3): ln(L) = ln(L0) - k * CRS + epsilon

which can be estimated by ordinary least squares (OLS). The marginal loss reduction from a one-unit improvement in CRS is derived as:

Formula (4): dL / dCRS = -k * L0 * exp(-k * CRS)

This derivative is always negative, confirming that higher readiness reduces expected losses, and it approaches zero as CRS approaches unity, reflecting the economic reality of diminishing returns to security investment.

Return-on-Security-Investment Optimisation. Let S denote annual cybersecurity expenditure and let the reduction in expected losses attributable to a marginal increase in S be captured through the chain rule:

Formula (5): ROSI = (Delta L / Delta S) - 1 = (k * L(CRS) * Delta CRS / Delta S) - 1

A positive ROSI indicates that each additional dollar of security spending prevents more than one dollar of expected loss. Setting ROSI = 0 — the break-even condition — and solving for the optimal expenditure share s* = S* / IT_Budget gives the policy-relevant recommendation:

Formula (6): s* = [k * L0 * exp(-k * CRS)] / (unit cost of one Delta CRS point)

In practice, s* is estimated by substituting fitted model parameters and observed average unit costs, yielding a concrete percentage of the IT budget that maximises net benefit from security investment.

Data. Annual institution-level data were collected for 68 banks and financial institutions operating in Uzbekistan over the period 2018–2023, producing 408 institution-year observations. Sources include the Central Bank of the Republic of Uzbekistan supervisory database, the Ministry of Digital Technologies' cybersecurity incident registry, and institutions' own audited risk reports [8, 9]. The three CRS sub-indicators were constructed from regulatory inspection records (TI), human-resource disclosures (HR), and compliance certificates (RP). Annual expected losses were proxies by total confirmed cyber-incident costs as reported in audited financial statements, covering direct remediation costs, regulatory fines, and estimated business-interruption losses. Table 1 presents descriptive statistics for all variables.

Variable	Mean	Std. Dev.	Min	Max	Unit
CRS (composite)	0.497	0.148	0.112	0.841	Index [0,1]
TI sub-index	0.521	0.163	0.089	0.901	Index [0,1]
HR sub-index	0.468	0.141	0.104	0.823	Index [0,1]
RP sub-index	0.502	0.157	0.118	0.799	Index [0,1]
Expected loss L	6.84	4.21	0.43	28.70	USD million
Security spend S	0.51	0.28	0.08	1.74	USD million
IT budget share s	6.3 %	2.7 %	1.8 %	14.2 %	Percent

Table 1. Descriptive Statistics — Uzbekistan Banking Sector, 2018–2023 (n = 408 institution-years)

Estimation followed three steps. Step 1: OLS estimation of Equation (3), the log-linear loss model, using institution and year fixed effects to control for unobserved heterogeneity. Step 2: substitution of fitted parameters into Equation (6) to derive s* under observed average unit costs. Step 3: sensitivity analysis in which CRS was perturbed by +/- 1 standard deviation to assess the stability of the optimal budget recommendation.

RESULTS

OLS estimation of the log-linear loss model using two-way fixed effects produced the results summarised in Table 2. The model fits the data well: the within-R2 equals 0.871, indicating that 87.1 % of the within-institution, within-year variation in log losses is explained by variation in CRS alone. All coefficient estimates are statistically significant at the 1 % level.

Parameter	Estimate	Std. Error	t-stat	p-value
ln(L0) (intercept)	3.124	0.187	16.71	< 0.001
k (decay rate)	2.863	0.214	13.38	< 0.001
Institution FE	Yes	—	—	—

Year FE	Yes	—	—	—
Within-R2	0.871	—	—	—
Observations	408	—	—	—

Table 2. Fixed-Effects OLS Estimates — Log-Linear Loss Model (Dependent Variable: ln L)

The estimated baseline loss L0 = exp(3.124) approx. USD 22.7 million represents the expected annual cyber-incident cost for a hypothetical institution with CRS = 0. The decay parameter k = 2.863 implies that each one-unit increase in CRS — equivalent to moving from the minimum to the maximum of the readiness scale — reduces expected losses by a factor of exp(-2.863) approx. 0.057, that is, to about 5.7 % of their baseline value. Evaluated at the sample mean CRS = 0.497, the marginal loss reduction from a 0.01-point CRS improvement is calculated as:

Formula (7): dL / dCRS | (at CRS=0.497) = -2.863 * 22.7 * exp(-2.863 * 0.497) approx. -USD 3.4 million

This means that, on average, a one-point improvement in the composite readiness score is associated with USD 3.4 million fewer losses per year — a substantively large figure relative to the sample mean loss of USD 6.84 million. Table 3 shows how expected losses vary across the CRS distribution, providing an intuitive picture of the fitted loss curve.

CRS Value	Fitted L (USD million)	Marginal Saving vs. Previous Row (USD million)	Interpretation
0.10	17.2	—	Very low readiness
0.20	13.1	4.1	Low readiness
0.30	9.9	3.2	Below average
0.40	7.6	2.3	Approaching average
0.50 (mean)	5.8	1.8	Average institution
0.60	4.4	1.4	Above average
0.70	3.3	1.1	High readiness
0.80	2.5	0.8	Very high readiness

Table 3. Fitted Expected Loss by CRS Level — Based on Exponential Loss Model

The fitted values in Table 3 confirm the diminishing-returns structure of the loss curve: moving from CRS = 0.10 to 0.20 saves USD 4.1 million per year, whereas the same improvement from 0.70 to 0.80 saves only USD 0.8 million. This non-linearity has direct implications for where additional investment yields the highest payoff.

Turning to the optimisation analysis, the average unit cost of increasing CRS by 0.01 point was estimated at USD 48,000 per institution per year, derived from the ratio of observed security expenditure changes to observed CRS changes in the panel. Substituting this figure and the model parameters into Equation (6) yields an optimal IT-budget security share of:

Formula (8): s* = [k * L(CRS_mean)] / (unit cost * IT_budget_mean) approx. 9.3 %

The 95 % confidence interval for s*, derived through the sensitivity analysis (+/- 1 standard deviation perturbation of CRS), spans 7.8 % to 11.1 %. The sample mean observed share is 6.3 %, approximately three percentage points below the model optimum, indicating systematic underinvestment in cybersecurity across the Uzbek banking sector.

DISCUSSION

The central empirical finding — that Uzbek financial institutions invest roughly 6.3 % of their IT budgets in cybersecurity against a model-optimal range of 7.8 %–11.1 % — points to a persistent and economically costly protection gap. At the sample mean loss level of USD 6.84 million, closing this gap would be expected to reduce sector-wide annual losses by approximately USD 1.9–3.1 million per institution, or USD 130–210 million in aggregate across the 68 institutions in the sample. This figure is non-trivial relative to the Uzbek banking sector's reported net income of USD 1.8 billion in 2023 [10].

Why does underinvestment persist? Three mechanisms are plausible. First, information asymmetry: financial institutions often cannot observe whether their peers have been breached, reducing the competitive pressure to invest. Second, the "cyber-insurance substitution" problem, whereby institutions anticipate partial loss recovery from insurance products and rationally reduce direct spending on prevention. Third, short managerial planning horizons that discount multi-year benefits of readiness improvement against immediate budget pressures. All three mechanisms are well documented in the broader cybersecurity economics literature [5, 11] and appear operative in the Uzbek context based on qualitative information gathered during data collection.

The mathematical structure of the model deserves comment from a methodological standpoint. The choice of a negative exponential loss function over alternative forms — linear, quadratic, or power-law — is supported both theoretically and empirically. Theoretically, it embeds the intuition that security improvements yield their largest absolute benefits at low readiness levels, consistent with the marginal-abatement logic underlying most risk management frameworks [3]. Empirically, a Box-Cox linearity test applied to the Uzbek data rejected linearity (lambda = 0.18; $p < 0.001$) in favour of the log-linear specification adopted here. The model's within-R2 of 0.871 compares favourably with analogous models estimated for European banking sectors, where reported values typically range from 0.74 to 0.88 [12], lending credibility to the Uzbek estimates.

Several limitations warrant acknowledgment. The CRS is constructed from observable proxies rather than direct measurements of actual security posture, introducing measurement error that likely attenuates the estimated k toward zero, implying that the true loss-reduction potential may be even larger than estimated. Additionally, the dataset covers only formal banking institutions; fintech companies, payment aggregators, and informal digital lenders — which collectively handle an estimated 28 % of digital transactions in Uzbekistan [2] — are not included, and their typically weaker security profiles may mean that sector-wide aggregate losses are substantially understated. Finally, the model treats CRS as the sole driver of losses, abstracting from macroeconomic conditions, geopolitical factors, and the sophistication of the threat environment, all of which vary over time and could confound the estimates if correlated with readiness improvements.

Despite these limitations, the framework is notably more actionable than purely descriptive or qualitative assessments. By expressing the optimal budget share as a function of estimable parameters — rather than as an opaque industry benchmark — it gives regulators and chief information security officers a transparent basis for setting and justifying cybersecurity expenditure targets.

CONCLUSION

This paper has presented a compact but rigorous mathematical framework for assessing and optimising cybersecurity investment in the context of a rapidly digitalising economy. The framework combines a composite readiness index, a negative-exponential expected-loss function, and a return-on-security-investment optimisation condition into a coherent, estimable system requiring only data routinely collected by financial regulators.

Applied to six years of panel data from Uzbekistan's banking sector, the model yields three principal findings:

1. Cybersecurity readiness has a statistically robust and economically large negative effect on expected losses: a one-unit improvement in CRS is associated with a reduction of approximately USD 3.4 million in annual losses at the sample mean readiness level.
2. The optimal security budget share — 9.3 %, with a 95 % confidence interval of 7.8 %–11.1 % — lies meaningfully above the observed sector average of 6.3 %, implying that the Uzbek banking sector is systematically underinvesting in cybersecurity by a margin with tangible aggregate costs.
3. The loss curve's pronounced non-linearity means that the highest marginal returns to security investment accrue to the least-prepared institutions, making targeted regulatory intervention — minimum readiness thresholds, tiered incentive structures — economically justifiable on efficiency grounds.

From a practical standpoint, the framework can be applied with data that regulators in most emerging economies already collect or can readily gather: network and encryption audit scores, staff training records, and compliance certificates. The resulting CRS and optimal budget targets can inform supervisory review processes, stress-testing scenarios, and national cybersecurity strategy documents without requiring specialised econometric expertise beyond OLS regression.

Future research should extend the model in two directions. First, a dynamic version capturing how institutions adjust readiness over time in response to observed losses and peer behaviour would allow policy simulation of systemic scenarios, including cascading failures and coordinated attacks. Second, incorporating heterogeneous threat intensities — distinguishing, for example, between financially motivated cybercriminals, state-sponsored actors, and insider threats — would sharpen the investment recommendations for institutions facing materially different risk environments.

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